Computer Programming

Introduction. Recursion

Marius Minea

marius@cs.upt.ro

http://cs.upt.ro/~marius/curs/cp/

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The C programming language

developed in 1972 at *AT&T Bell Laboratories* by Dennis Ritchie
together with the UNIX operating system and its tools
(C first developed under UNIX, then UNIX was rewritten in C)
Brian Kernighan, Dennis Ritchie: The C Programming Language (1978)

Mature language, but still evolving
ANSI C standard, 1989 (American National Standards Institute)
then ISO 9899 standard (versions: C90, C99, C11 - current)

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Why use C?
versatile: direct access to data representation, freedom in working with memory, good hardware interface
mature, large code base (libraries for many purposes)
efficient: good compilers that generate compact, fast code

WARNING: very easy to make *errors* !

Computations, functions, and programs

A program

reads input data

processes them (through (mathematical) *computations*)

writes (produces) *results*

Computations, functions, and programs

A program
- *reads input data*
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- *writes* (produces) *results*

In mathematics, computations are expressed by *functions*:
- we *know* predefined functions (sin, cos, etc.)
- we *define* new functions (for the given problem)
- we *combine* functions into more complex computations

In programming, we use functions in a similar way.

Functions are the core of programming

Programs are structured into functions (methods, procedures)

Splitting into functions helps *manage complexity* !

Functions can be *reused*, making development efficient.

Functions are core to defining what is computable
(recursive functions, lambda calculus, functional programming)

Functions in mathematics and C

Squaring for integers:

$sqr : \mathbb{Z} \to \mathbb{Z}$

$sqr(x) = x \cdot x$

function type | function name | parameter type and name

```
int sqr(int x)
{
  return x * x;
}
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A function *definition* contains:

the function *header*, specifying: the type (range) of function values (int), function name (`sqr`) and parameters (the integer `x`)

the function *body*, within `{ }`: here, the `return` *statement*, with an *expression* that gives the function value from its parameters

There are precise *rules* for writing in the language (the *syntax*):

language elements are written in a given *order*;

separators are used to precisely delimit them: `( )` `;` `{ }`

Another function

Squaring for *reals*:

$$sqrf : \mathbb{R} \rightarrow \mathbb{R}$$

$$sqrf(x) = x \cdot x$$

```
float sqrf(float x)
{
  return x * x;
}
```

Another function domain and range (reals) $\Rightarrow$ a different function
   even the `*` operator is now defined on a different set (type)
To distinguish it from `sqr` in the same program, it needs a different name.

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To distinguish it from `sqr` in the same program, it needs a different name.

`int` and `float` denote *types*.

A *type* is a *set of values* together with a *set of operations* allowed for these values.

For reals, it is preferable to use the type `double` (double precision)
(used by library functions: `sin`, `cos`, `exp`, etc.)

Integers and reals

Numeric types differ in C and mathematics.

In mathematics: $\mathbb{Z} \subset \mathbb{R}$, both are infinite, $\mathbb{R}$ is uncountable.

In C: `int`, `float`, `double` are finite (have limited range);
reals have finite precision.

Important to remember this! (overflows, precision loss)

The type of numeric *constants* depends on their writing

- `2` is an integer, `2.0` is a real
- scientific notation for reals: `1.0e-3` instead of `0.001`
- writing `1.0` or `1.` is equivalent, same for `0.1` and `.1`

Mathematical operators

`+ - * /`

Multiplication is written explicitly !
we can't write $2x$, but `2 * x` (or `x * 2`)

Some operators have different meanings (and results!) for integers and reals:

Integer division has an *integer result* !!! (division with remainder)
`7 / 2` is `3`, but `7.0 / 2.0` is `3.5`
`-7 / 2` is −3, likewise `-(7 / 2)`
(integer division truncates towards zero)

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The *modulo* operator % is only defined for integers.

`9 / 5` = `1`	`9 % 5` = `4`	`9 / -5` = `-1`	`9 % -5` = `4`
`-9 / 5` = `-1`	`-9 % 5` = `-4`	`-9 / -5` = `1`	`-9 % -5` = `-4`

The sign of the remainder is the same as the sign of the dividend.

Rule for integer division: `a` = `a / b * b + a % b`

Some terminology

Keywords: have a predefined meaning (cannot be changed)
Examples: statements (`return`), types (`int`, `float`, `double`)

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An identifier is a sequence of characters comprised of letters (upper and lower case), underscore _ and digits which does not start with a digit and is not a keyword.
Examples: `x3, a12_34, _exit, main, printf, int16_t`

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Constants
integer: `-2`; floating point: `3.14`; character: `'a'`, string: `"a"`

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Punctuation signs, with various meanings:

- `*` is an operator
- `;` terminates a statement
- parantheses `( )` around an expression or function parameters
- braces `{ }` group declarations or statements

Functions with several parameters

Example: the discriminant of a quadratic equation:
$a \cdot x^2 + b \cdot x + c = 0$

```
float discrim(float a, float b, float c)
{
  return b * b - 4 * a * c;
}
```

Between the parantheses `( )` of the function header there can be arbitrary comma-separated parameters, each with its own type.

Function call (function evaluation)

So far, we have only *defined* functions, without using them.

The value of a function can be *used* in an expression.

Syntax: like in mathematics: *function(param, param,* $\cdots$ *, param)*

Example: in the discriminant, we could use the `sqrf` function:

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return sqrf(b) - 4 * a * c;
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Or, using the previously defined `sqr` function we can define:

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int cube(int x)
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  return x * sqr(x);
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}
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IMPORTANT: In C, any identifier must be *declared before use*
(we must know what it represents, including its type)
⇒ The above examples assume that `sqrf` and `sqr` are defined
before `discrim` and `cube` respectively in the program.

A first C program

```c
int main(void)
{
  return 0;
}
```

The smallest program: it does not do anything!

Any program contains the *main* function and is executed by calling it at program start. In `main`, other functions may be called.

Here, `main` does not have any parameters (`void`)
  `void` is a keyword for the empty type (without any element)

`main` returns an integer, interpreted as exit status by the operating system:
$0 =$ successful termination, $\neq 0$ is an error code

A commented program

```c
/* This is a comment */
int main(void) // comment to end of line
{
  /* This is a comment spanning several lines
    usually, the program code would be here */
  return 0;
}
```

Programs may contain comments, placed between /* and */
or starting with // until (and excluding) the end of the line
Comments are stripped by the preprocessor.
They have no effect on code generation or program execution.

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Programs *should be* commented
- so a reader can understand (including the writer, at a later time)
- as documentation (may specify functionality, restrictions, etc.)
- explain function parameters, result, local variables
- specify preconditions, postconditions, error behavior

Printing (writing)

```
#include <stdio.h>
int main(void)
{
  printf("hello, world!\n"); // prints a text
  return 0;
}
```

`printf` (from ”print formatted”): a standard library function

- is NOT a *statement* or a *keyword*
- is called here with one string parameter
- string constants are written with double quotes `" "`

`\n` denotes the newline character

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\n denotes the newline character

The first line is a *preprocessing directive*, it includes the stdio.h *header file* which contains the *declarations* of the standard input/output functions

Declaration = type, name, parameters: needed to use the function

Implementation (compiled object code): in a *library* which is linked at compile-time, loaded at execution time

Printing numbers

```
#include <math.h>
#include <stdio.h>
int main(void)
{
  printf("cos(0) = ");
  printf("%f", cos(0));
  return 0;
}
```

```
#include <stdio.h>
int sqr (int x) { return x * x; }
int main(void)
{
  printf("2 times -3 squared is ");
  printf("%d", 2 * sqr(-3));
  return 0;
}
```

To print the value of an expression, `printf` takes two arguments:

– a character string (format specifier):
  `%d` or `%i` (*decimal* integer), `%f` (*floating point*)

– the expression; type must be compatible with the specified one
  (programmer must check! compiler may warn or not)

Sequencing: in function, statements are executed in textual order

But: `return` statement ends function execution (no further statement is executed)

Printing

We cannot print a number like this: ~~`printf(5)`~~

We can write `printf("5")` but this means printing a *string*
(although the effect is the same: one character printed)

The first argument of `printf` must always be a string
with or without format specifiers (special characters)

Understanding how functions work

Two distinct things:

function *definition*: `int sqr(int x) { ... }`

function *call*: `sqr(2)`, `sqr(a)`, etc.

Function definitions use *names* (of parameters, variables, etc.)

Function calls work with *values* (2, the *value* of a, etc.)
(they do *not* compute with symbolic expressions)

Understanding the function call

This program computes $2^6 = (2 \cdot 2^2)^2$

```
#include <stdio.h>
int sqr(int x)
{
  printf("the square of %d is %d\n", x, x*x);
  return x * x;
}
int main(void)
{
  printf("2 to the 6th is %d\n", sqr(2 * sqr(2)));
  return 0;
}
```

What is the order of printed statements ?

```
the square of 2 is 4
the square of 8 is 64
2 to the 6th is 64
```

C uses call by value

In C, function arguments are passed *by value*.

all function arguments are *evaluated* (their value is computed)
values are assigned to the *formal parameters* (names from the function header)
then, function is *called* and executes with these values

This type of argument passing is named *call by value*.

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The program starts executing `main`. The first statement:

```
printf("2 to the 6th is %d\n", sqr(2 * sqr(2)));
```

Before doing the call, `printf` needs the *values of its arguments*

first argument: the value is known (a *string constant*)

second argument: need to call `sqr(2 * sqr(2))`

BUT: the outer `sqr` also needs the value of its argument
`2 * sqr(2)` ⇒ need to call `sqr(2)` first

⇒ call order: first `sqr(2)`, then `sqr(8)`, then `printf`

Errors in understanding function evaluation

C does NOT do the following (other languages might...)

Functions do NOT start execution without computer arguments
`printf` would print `2 to the 6th is` , then need the value
it would call the outer `sqr` that writes `the square of`,
then would need `x`
it would call `sqr(2)`, write `the square of 2 is 4`, return 4,
etc.

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then would need `x`
it would call `sqr(2)`, write `the square of 2 is 4`, return 4,
etc.

Function parameters are NOT substituted with expressions
`printf` would call the outer `sqr` with the *expression* `2 * sqr(2)`
`sqr(2)` would be called twice for `(2*sqr(2))*(2*sqr(2))`

⇒ in C, a function computes with *values*, never with *expressions*

Functions defined by cases

$$abs : \mathbb{Z} \to \mathbb{Z} \qquad abs(x) = \begin{cases} x & x \geq 0 \\ -x & \text{otherwise } (x < 0) \end{cases}$$

The function value is not given by a *single* expression, but by one of two different expressions (x or -x), depending on a condition ($x \geq 0$)

$\Rightarrow$ need a language construct that to *decide* which expression to evaluate, based on a *condition* (true/false)

The conditional operator ? :

Syntax of *conditional expression*: *condition* `?` *expr1* `:` *expr2*
– if the condition is true, only *expr1* is evaluated, its value becomes the result of the entire expression
– if the condition is false, only *expr2* is evaluated and its value becomes the value of the expression

```c
int abs(int x)
{
  return x >= 0 ? x : -x;     // unary minus operator
}
```

Comparison operators: `==` (equality), `!=` (different), <, <=, >, >=

IMPORTANT! The equality test in C is `==` and not simple = !!!

Note: `abs` exists as standard function, declared in `stdlib.h`

Functions defined by several cases

$$sgn : \mathbb{Z} \to \{-1, 0, 1\} \qquad sgn(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$

The conditional operator has only *one* condition, and *two* branches
But: either of the expressions can be arbitrarily complex
$\Rightarrow$ must decompose the decision based on the value of x
$\Rightarrow$ *decompose into smaller subproblems*: key in problem solving

We rewrite the function with a single decision at any given point:

$$sgn(x) = \begin{cases} \text{if } x < 0 & -1 \\ \text{else } (x \geq 0) & \begin{cases} \text{if } x = 0 & 0 \\ \text{else } (x > 0) & 1 \end{cases} \end{cases}$$

Writing the case-based function in C

$$sgn(x) = \begin{cases} \text{if } x < 0 & -1 \\ \text{else } (x \geq 0) & \begin{cases} \text{if } x = 0 & 0 \\ \text{else } (x > 0) & 1 \end{cases} \end{cases}$$

```
int sgn (int x)
{
  return x < 0 ? -1
    : x == 0 ? 0 : 1;
}
```

We can group arbitrarily many conditional operators `?` `:`
expr1 and *expr2* can be in turn conditional expressions
A correctly written expression has a `:` for any `?`
(think of `:` as linking a *pair* of answers)

Decomposing into simpler problems

The minimum of two numbers is easily written:

```
double min2(double x, double y)
{
  return x < y ? x : y;
}
```

For the minimum of *three* numbers, the comparisons multiply:

$$min3(x, y, z) = \begin{cases} \text{if } x < y & \begin{cases} \text{if } x < z & \mathbf{x} \\ \text{else } (x \geq z) & \mathbf{z} \end{cases} \\ \text{else } (x \geq y) & \begin{cases} \text{if } y < z & \mathbf{y} \\ \text{else } (y \geq z) & \mathbf{z} \end{cases} \end{cases}$$

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$$

We notice the structure of `min2` is repeated $\Rightarrow$ can do it simpler:
The result is the minimum between the minimum of the first two numbers and the third. $\Rightarrow$ just apply `min2` twice!

```
double min3(double x, double y, double z)
{
  return min2(min2(x, y), z); // or min2(x, min2(y,z))
}
```

Recursion

Recursion: definition, examples

From mathematics, we know recurrence relations for *sequences*:

arithmetic sequence: $\begin{cases} x_0 = b & \text{(i.e.: } x_n = b \text{ for } n = 0) \\ x_n = x_{n-1} + r & \text{for } n > 0 \end{cases}$

Example: $1, 4, 7, 10, 13, \ldots$ ($b = 1$, $r = 3$)

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geometric sequence: $\begin{cases} x_0 = b & \text{(i.e.: } x_n = b \text{ for } n = 0\text{)} \\ x_n = x_{n-1} \cdot r & \text{for } n > 0 \end{cases}$

Example: $3, 6, 12, 24, 48, \ldots$ ($b = 3$, $r = 2$)

x_n is not computed *directly*, but *step by step*, using x_{n-1}.

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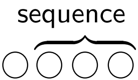
A notion is *recursive* if it is *used in its own definition*.

Exercise: write recurrences for: C_n^k, Fibonacci sequence, ...

Recursion: definition, examples

Recursion is fundamental in computer science:
it reduces a problem to a simpler case of the *same* problem

objects: a *sequence* is

- a single element 
- an element followed by a *sequence* 

e.g. word (sequence of letters); number (sequence of digits)

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actions: a *path* is

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e.g. traversing a path in a graph

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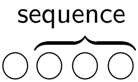

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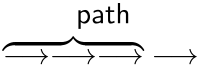

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An *expression*:

- number (7)
- identifier (x)
- *expression* + *expression*
- *expression* – *expression*
- (*expression*), etc

Example: power function

$$x^n = \begin{cases} 1 & n = 0 \\ x \cdot x^{n-1} & \text{otherwise } (n > 0) \end{cases}$$

```
#include <stdio.h>
double pwr(double x, unsigned n)
{
 return n==0 ? 1 : x * pwr(x, n-1);
}
int main(void)
{
  printf("-2 raised to 3 = %f\n", pwr(-2.0, 3));
  return 0;
}
```

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}
```

`unsigned`: type of nonnegative integers (natural numbers)

The *header* of `pwr` is a *declaration* of the function
so it can be used in its own function body (recursive call)

Even if we write `pwr(-2, 3)`, -2 (int) will be *converted* to float
(the type declared for each parameter is known)

The mechanism of a recursive call

The `pwr` function does two computations:

– a *test* (`n == 0` ? *base case* ?) if so, return `1`

– else, a multiply; the right operand requires a *new recursive call*

```
pwr(5, 3)
      call↓ ↑125
        5 * pwr(5, 2)
              call↓ ↑25
                5 * pwr(5, 1)
                      call↓ ↑5
                        5 * pwr(5, 0)
                              call↓ ↑1
                                1
```

The mechanism of a recursive call

In the recursive computation of the power function:

Every call makes *a new call*, until the base case it reached

Every call executes *the same code*, but with *other data*
(own values for parameters)

When reaching the base case, all started calls are still *unfinished*
(each has to perform the multiplication with the result of the call)

Returning is done *in opposite order* of the calls
(call with exponent 0 returns, then the one with exponent 1, etc.)

Recursion: power by repeated squaring

Recursion = reduction to a *simpler* case of the *same* problem

Base case is simple enough for direct computation
(can / need no longer be reduced)

$$x^n = \begin{cases} 1 & n = 0 \\ (x^2)^{n/2} & n > 0 \text{ even} \\ x \cdot (x^2)^{n/2} & n > 0 \text{ odd} \end{cases}$$

```
double pow2(double x, unsigned n)
{
  return n == 0 ? 1
    : n % 2 == 0 ? pow2(x*x, n/2)
    : x * pow2(x*x, n/2);
}
```

Let's follow the recursive calls

```
#include <stdio.h>

double pow2(double x, unsigned n)
{
  printf("base %f exponent %u\n", x, n);
  return n == 0 ? 1
    : n % 2 == 0 ? pow2(x*x, n/2)
    : x * pow2(x*x, n/2);
}
int main(void)
{
  printf("5 to the 6th = %f\n", pow2(5, 6));
  return 0;
}
```

Each call halves the exponent $\Rightarrow 1 + \lceil \log_2 n \rceil$ calls

`pow2(5, 6)` $\rightarrow$ `pow2(25, 3)` $\rightarrow$ `pow2(625, 1)`

How to use recursion

Recursion solves a problem by reducing it to a simpler case of the same problem.

To use recursion, we must express the problem as a *function*

  things given/known to the function are *parameters*

    (index of recursive sequence; problem size; etc.)

  the answer to the problem is the function *result*

Sometimes, the problem asks to *produce an effect* (print) rather than compute a result.

Block statements and sequencing

A function body may have several statements *in sequence*

```
{
  printf("This is a line\n");
  printf("Line 2: ");
  printf("cos(0)=%f\n", cos(0));
  return 0;
}
```

{
statement
...
statement
}

Function returns on reaching closing brace OR `return` statement.

More generally, a *block* (compound statement) can appear in place of any statement.

This is an example of *recursion* in the *definition of statements*:

statement ::= `return` $expression_{optional}$;
$expression_{optional}$; (incl. function call)
{ *statement* ... *statement* }

The `if` statement

Conditional operator `? :` selects from two *expressions* to evaluate

Conditional statement selects between two *statements* to execute

Syntax:

`if` (*expression*)
  statement1
`else`
  statement2

or

`if` (*expression*)
  statement1

Effect:

If the expression is *true* (nonzero) *statement1* is executed,
else *statement2* is executed (or nothing, if the latter is missing)

Each branch has only *one* statement. If several statements are needed, these must be grouped in a *compound statement* `{ }`

The *parantheses* ( ) around the condition are mandatory.

Example with the if statement

Printing roots of a quadratic equation:

```
void printsol(double a, double b, double c)
{
  double delta = b * b - 4 *a * c;
  if (delta >= 0) {
    printf("root 1: %f\n", (-b-sqrt(delta))/2/a);
    printf("root 2: %f\n", (-b+sqrt(delta))/2/a);
  } else printf("no solution\n"); // puts("no solution");
}
```

Can rewrite the *conditional operator* `? :` using the `if` *statement*

```
int abs(int x)
{
  return x > 0 ? x : -x;
}
```

```
int abs(int x)
{
  if (x > 0) return x;
  else return -x;
}
```

Recursion: Fibonacci words

Fibonacci sequence: $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$ for $n > 1$
inefficient to do direct recursion (exercise: how many calls?)

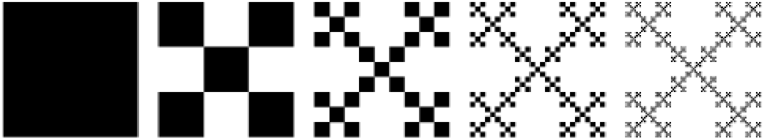
Can define Fibonacci words (strings):
$S_0 = 0, S_1 = 01, S_n = S_{n-1}S_{n-2}$
(formed by string *concatenation*)

Write a function that prints S_n
problem = function; effect = print; concatenation = ???

More recursion: fractals

Fractals are *self-similar* figures
(a part of the figure looks like the whole figure = recursion!)

Box fractal:



More recursion: fractals

Fractals are *self-similar* figures

(a part of the figure looks like the whole figure = recursion!)

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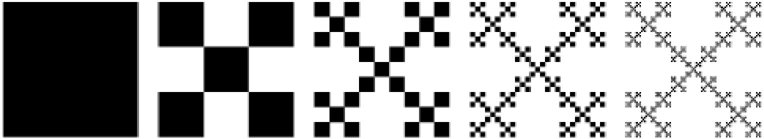

What is the base case?

What defines a part of the figure?

http://mathworld.wolfram.com/BoxFractal.html