Computer Programming

Introduction. Recursion

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Course goals

Learn programming fundamentals

no prior knowledge needed

for those who know, hopefully learn more

Know one language well

imperative programming in C

some insight into alternatives

Write clean, correct, secure code

handle errors

test your code

think of corner cases

The C programming language

developed in 1972 at *AT&T Bell Laboratories* by Dennis Ritchie
together with the UNIX operating system and its tools
(C first developed under UNIX, then UNIX was rewritten in C)
Brian Kernighan, Dennis Ritchie: The C Programming Language (1978)

Mature language, but still evolving
ANSI C standard, 1989 (American National Standards Institute)
then ISO 9899 standard (versions: C90, C99, C11 - current)

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Why use C?
versatile: direct access to data representation, freedom in working with memory, good hardware interface
mature, large code base (libraries for many purposes)
efficient: good compilers that generate compact, fast code

WARNING: very easy to make *errors* !

Computations, functions, and programs

A program

reads *input data*

processes them – through (mathematical) *computations*

writes (produces) *results*

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In mathematics, computations are expressed by *functions*:
we *know* predefined functions (sin, cos, etc.)
we *define* new functions (for the given problem)
we *combine* functions into more complex computations

In programming, we use functions in a similar way.

Functions are the core of programming

Programs are *structured* into functions (methods, procedures)

Splitting into functions helps *manage complexity*
NOT one huge piece of code

Functions can be *reused*, making development efficient

Functions are core for the *functional programming* paradigm
computation is function *evaluation*, not ~~assignment~~

Functions are core to defining what is *computable*
(recursive functions, lambda calculus)

Functions in mathematics and C

Squaring for integers:

$sqr : \mathbb{Z} \to \mathbb{Z}$

$sqr(x) = x \cdot x$

function type | function name | parameter type and name

```
int sqr(int x)
{
  return x * x;
}
```

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A function *definition* contains:

the function *header*, specifying: the type (range) of function values (int), function name (`sqr`) and parameters (the integer `x`)

the function *body*, within `{ }`: here, the `return` *statement*, with an *expression* that gives the function value from its parameters

There are precise *rules* for writing in the language (the *syntax*):

language elements are written in a given *order*;

separators are used to precisely delimit them: `( )` `;` `{ }`

Functions in C vs. other languages

concrete syntax: detail (keywords, punctuation)

vs.

abstract syntax: essence (language elements/concepts)

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```
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```

Essence:

names: function, parameter(s)

types: of parameter(s) and return value

cannot omit (some languages: can infer types)

one precise type (some languages: polymorphism, overloading)

expression (what is computed)

Details (concrete syntax):

`return` keyword, punctuation: `{ ; }`, order (types first)

Another function

Squaring for *reals*:

$sqrf : \mathbb{R} \rightarrow \mathbb{R}$

$sqrf(x) = x \cdot x$

```
float sqrf(float x)
{
  return x * x;
}
```

Another function domain and range (reals) ⇒ a different function
even the `*` operator is now defined on a different set (type)

Need different name to distinguish from `sqr` in the same program

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`int` and `float` denote *types*

A *type* is a *set of values*
together with a *set of operations* allowed for these values.

For reals, it is preferable to use the type `double` (double precision)
(used by library functions: `sin`, `cos`, `exp`, etc.)

Integers and reals

Numeric types differ in C and mathematics.

In math: $\mathbb{Z} \subset \mathbb{R}$, both are *infinite*, $\mathbb{R}$ is dense/uncountable.

In C: `int`, `float`, `double` are *finite!*
both have *limited range*, reals have *finite precision*

Important to remember this! (overflows, precision loss)
default math functions use `double`, you should too!

The type of numeric *constants* depends on their writing

2 is an integer, `2.0` is a real

scientific notation for reals: `1.0e-3` instead of `0.001`

writing `1.0` or `1.` is equivalent, same for `0.1` and `.1`

Mathematical operators

`+ - * /`

Multiplication must be written explicitly !
we can't write $2x$, but `2 * x` (or `x * 2`)

Some operators have different meanings for integers and reals
and different results!

Integer division has an *integer result* !!! (division with remainder)

`7 / 2` is `3`, but `7.0 / 2.0` is `3.5`

`-7 / 2` is `-3`, likewise `-(7 / 2)`

(integer division truncates towards zero)

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The *modulo* operator `%` is only defined for integers.

`9 / 5 = 1`	`9 % 5 = 4`	`9 / -5 = -1`	`9 % -5 = 4`
`-9 / 5 = -1`	`-9 % 5 = -4`	`-9 / -5 = 1`	`-9 % -5 = -4`

Rule for integer division: `a = a / b * b + a % b`

$\Rightarrow$ sign of remainder is same as sign of dividend.

Some terminology

Keywords: have a predefined meaning (cannot be changed)
Examples: statements (`return`), types (`int`, `float`, `double`)

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Examples: `x3, a12_34, _exit, main, printf, int16_t`

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Constants
integer: `-2`; floating point: `3.14`; character: `'a'`, string: `"a"`

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Punctuation signs, with various meanings:

- `*` is an operator
- `;` terminates a statement
- parantheses `( )` around an expression or function parameters
- braces `{ }` group declarations or statements

Functions with several parameters

Example: the discriminant of a quadratic equation:
$a \cdot x^2 + b \cdot x + c = 0$

```
float discrim(float a, float b, float c)
{
  return b * b - 4 * a * c;
}
```

Between the parantheses `( )` of the function header there can be arbitrary comma-separated parameters, each with its own type.
must give type for each parameter, even if types are the same

Function call (function evaluation)

So far, we have only *defined* functions, without using them.
The value of a function can be *used* in an expression.
Syntax: like in mathematics: *function(param, param,* $\cdots$ *, param)*
Example: in the discriminant, we could use the `sqrf` function:

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return sqrf(b) - 4 * a * c;
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Or, using the previously defined `sqr` function we can define:

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int cube(int x)
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  return x * sqr(x);
}
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IMPORTANT: In C, any identifier must be *declared before use*
(we must know what it represents, including its type)
⇒ The above examples assume that `sqrf` and `sqr` are defined
before `discrim` and `cube` respectively in the program.

A first C program

```c
int main(void)
{
  return 0;
}
```

The smallest program: it does not do anything!

Any program contains the *main* function and is executed by calling it at program start. In `main`, other functions may be called.

Here, `main` does not have any parameters (`void`)
`void` is a keyword for the empty type (without any element)

`main` returns an int, interpreted as exit status by operating system:
$0 =$ successful termination, $\neq 0$ is an error code

`return 0;` at the end of main is optional (if end brace is reached, 0 is returned by default; still most programs have it explicit).

A commented program

```
/* This is a comment */
int main(void) // comment to end of line
{
  /* This is a comment spanning several lines
    usually, the program code would be here */
  return 0;
}
```

Programs may contain comments, placed between /* and */ or starting with // until (and excluding) the end of the line
Comments are stripped by the preprocessor.
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Programs *should be* commented
- so a reader can understand (including the writer, at a later time)
- as documentation (may specify functionality, restrictions, etc.)
- explain function parameters, result, local variables
- specify preconditions, postconditions, error behavior

Printing (writing)

```c
#include <stdio.h>
int main(void)
{
  printf("hello, world!\n"); // prints a text
  return 0;
}
```

printf (from "print formatted"): a standard library function

- is NOT a *statement* or a *keyword*
- is called here with one string parameter
- string constants are written with double quotes " "

\n denotes the newline character

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`\n` denotes the newline character

The first line is a *preprocessing directive*, it includes the `stdio.h` *header file* which contains the *declarations* of the standard input/output functions

Declaration = type, name, parameters: needed to use the function

Implementation (compiled object code): in a *library* which is linked at compile-time, loaded at execution time

Printing numbers

```
#include <math.h>
#include <stdio.h>
int main(void)
{
  printf("cos(0) = ");
  printf("%f", cos(0));
  return 0;
}
```

```
#include <stdio.h>
int sqr (int x) { return x * x; }
int main(void)
{
  printf("2 times -3 squared is ");
  printf("%d", 2 * sqr(-3));
  return 0;
}
```

To print the value of an expression, `printf` takes two arguments:

- a character string (format specifier):
  `%d` or `%i` (*decimal* integer), `%f` (*floating point*)
- the expression; type must be compatible with the specified one
  (programmer must check! compiler may warn or not)

Sequencing: in function, statements are executed in textual order
But: `return` statement ends function execution (no further statement is executed)

Printing

We cannot print a number like this: ~~`printf(5)`~~

We can write `printf("5")` but this means printing a *string*
(although the effect is the same: one character printed)

The first argument of `printf` must always be a string
with or without format specifiers (special characters)

Understanding how functions work

Two distinct things:

function *definition*: `int sqr(int x) { ... }`

function *call*: `sqr(2)`, `sqr(a)`, etc.

Function definitions use *names* (of parameters, variables, etc.)

Function calls work with *values* (2, the *value* of a, etc.)
(they do *not* compute with symbolic expressions)

Understanding the function call

This program computes $2^6 = (2 \cdot 2^2)^2$

```
#include <stdio.h>
int sqr(int x)
{
  printf("the square of %d is %d\n", x, x*x);
  return x * x;
}
int main(void)
{
  printf("2 to the 6th is %d\n", sqr(2 * sqr(2)));
  return 0;
}
```

What is the order of printed statements ?

```
the square of 2 is 4
the square of 8 is 64
2 to the 6th is 64
```

C uses call by value

In C, function arguments are passed *by value*.

all function arguments are *evaluated* (their value is computed)

values are assigned to the *formal parameters* (names from the function header)

then, function is *called* and executes with these values

This type of argument passing is named *call by value*.

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The program starts executing `main`. The first statement:

```
printf("2 to the 6th is %d\n", sqr(2 * sqr(2)));
```

Before doing the call, `printf` needs the *values of its arguments*

first argument: the value is known (a *string constant*)
second argument: need to call `sqr(2 * sqr(2))`
BUT: the outer `sqr` also needs the value of its argument
`2 * sqr(2)` ⇒ need to call `sqr(2)` first

⇒ call order: first `sqr(2)`, then `sqr(8)`, then `printf`

Errors in understanding function evaluation

C does NOT do the following (other languages might...)

Functions do NOT start execution without computer arguments
 `printf` would print `2 to the 6th is` , then need the value
 it would call the outer `sqr` that writes `the square of`,
then would need `x`
 it would call `sqr(2)`, write `the square of 2 is 4`, return 4,
etc.

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etc.

Function parameters are NOT substituted with expressions
`printf` would call the outer `sqr` with the *expression* `2 * sqr(2)`
`sqr(2)` would be called twice for `(2*sqr(2))*(2*sqr(2))`

$\Rightarrow$ in C, a function computes with *values*, never with *expressions*

Decision

Functions defined by cases

$$abs : \mathbb{Z} \to \mathbb{Z} \qquad abs(x) = \begin{cases} x & x \geq 0 \\ -x & \text{otherwise } (x < 0) \end{cases}$$

The function value is not given by a *single* expression, but by one of two different expressions (x or -x), depending on a condition ($x \geq 0$)

$\Rightarrow$ need a language construct that to *decide* which expression to evaluate, based on a *condition* (true/false)

The conditional operator ? :

Syntax of *conditional expression*: *condition* `?` *expr1* `:` *expr2*
– if the condition is true, only *expr1* is evaluated, its value becomes the result of the entire expression
– if the condition is false, only *expr2* is evaluated and its value becomes the value of the expression

```
int abs(int x)
{
  return x >= 0 ? x : -x;    // unary minus operator
}
```

Comparison operators: `==` (equality), `!=` (different), <, <=, >, >=

IMPORTANT! The equality test in C is `==` and not simple = !!!

Note: `abs` exists as standard function, declared in `stdlib.h`

Functions defined by several cases

$$sgn : \mathbb{Z} \rightarrow \{-1, 0, 1\} \qquad sgn(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$

The conditional operator has only *one* condition, and *two* branches

But: either of the expressions can be arbitrarily complex

$\Rightarrow$ must decompose the decision based on the value of x

$\Rightarrow$ *decompose into smaller subproblems*: key in problem solving

We rewrite the function with a single decision at any given point:

$$sgn(x) = \begin{cases} \text{if } x < 0 & -1 \\ \text{else } (x \geq 0) & \begin{cases} \text{if } x = 0 & 0 \\ \text{else } (x > 0) & 1 \end{cases} \end{cases}$$

Writing the case-based function in C

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```
int sgn (int x)
{
  return x < 0 ? -1
    : x == 0 ? 0 : 1;
}
```

We can group arbitrarily many conditional operators `?` `:`
expr1 and *expr2* can be in turn conditional expressions
A correctly written expression has a `:` for any `?`
(think of `:` as linking a *pair* of answers)

Decomposing into simpler problems

The minimum of two numbers is easily written:

```
double min2(double x, double y)
{
  return x < y ? x : y;
}
```

For the minimum of *three* numbers, the comparisons multiply:

$$min3(x, y, z) = \begin{cases} \text{if } x < y & \begin{cases} \text{if } x < z & \mathbf{x} \\ \text{else } (x \geq z) & \mathbf{z} \end{cases} \\ \text{else } (x \geq y) & \begin{cases} \text{if } y < z & \mathbf{y} \\ \text{else } (y \geq z) & \mathbf{z} \end{cases} \end{cases}$$

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We notice the structure of `min2` is repeated $\Rightarrow$ can do it simpler: The result is the minimum between the minimum of the first two numbers and the third. $\Rightarrow$ just apply `min2` twice!

```
double min3(double x, double y, double z)
{
  return min2(min2(x, y), z); // or min2(x, min2(y,z))
}
```

Recursion

Recursion: definition, examples

From mathematics, we know recurrence relations for *sequences*:

arithmetic sequence: $\begin{cases} x_0 = b & \text{(i.e.: } x_n = b \text{ for } n = 0) \\ x_n = x_{n-1} + r & \text{for } n > 0 \end{cases}$

Example: $1, 4, 7, 10, 13, \ldots$ $(b = 1,\ r = 3)$

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geometric sequence: $\begin{cases} x_0 = b & \text{(i.e.: } x_n = b \text{ for } n = 0) \\ x_n = x_{n-1} \cdot r & \text{for } n > 0 \end{cases}$

Example: $3, 6, 12, 24, 48, \ldots$ ($b = 3$, $r = 2$)

x_n is not computed *directly*, but *step by step*, using x_{n-1}.

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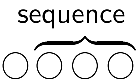
A notion is *recursive* if it is *used in its own definition*.

Exercise: write recurrences for: C_n^k, Fibonacci sequence, ...

Recursion: definition, examples

Recursion is fundamental in computer science:
it reduces a problem to a simpler case of the *same* problem

objects: a *sequence* is

- a single element 
- an element followed by a *sequence* 

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actions: a *path* is

- a step $\longrightarrow$
- a *path* followed by a step $\overbrace{\longrightarrow\longrightarrow\longrightarrow}^{\text{path}}\longrightarrow$

e.g. traversing a path in a graph

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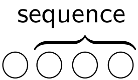

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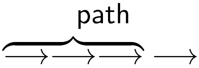

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An *expression*:

- number (7)
- identifier (x)
- *expression* + *expression*
- *expression* - *expression*
- (*expression*), etc

Example: power function

$$x^n = \begin{cases} 1 & n = 0 \\ x \cdot x^{n-1} & \text{otherwise } (n > 0) \end{cases}$$

```
#include <stdio.h>
double pwr(double x, unsigned n)
{
 return n==0 ? 1 : x * pwr(x, n-1);
}
int main(void)
{
  printf("-2 raised to 3 = %f\n", pwr(-2.0, 3));
  return 0;
}
```

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double pwr(double x, unsigned n)
{
 return n==0 ? 1 : x * pwr(x, n-1);
}
int main(void)
{
  printf("-2 raised to 3 = %f\n", pwr(-2.0, 3));
  return 0;
}
```

`unsigned`: type of nonnegative integers (natural numbers)

The *header* of `pwr` is a *declaration* of the function
so it can be used in its own function body (recursive call)

Even if we write `pwr(-2, 3)`, -2 (int) will be *converted* to float
(the type declared for each parameter is known)

The mechanism of a recursive call

The `pwr` function does two computations:
– a *test* (`n == 0` ? *base case* ?) if so, return 1
– else, a multiply; the right operand requires a *new recursive call*

```
pwr(5, 3)
      call↓ ↑125
         5 * pwr(5, 2)
               call↓ ↑25
                  5 * pwr(5, 1)
                        call↓ ↑5
                           5 * pwr(5, 0)
                                 call↓ ↑1
                                    1
```

The mechanism of a recursive call

In the recursive computation of the power function:

Every call makes *a new call*, until the base case it reached

Every call executes *the same code*, but with *other data*
(own values for parameters)

When reaching the base case, all started calls are still *unfinished*
(each has to perform the multiplication with the result of the call)

Returning is done *in opposite order* of the calls
(call with exponent 0 returns, then the one with exponent 1, etc.)